ДОМАШНЕЕ ЗАДАНИЕ № 4
(ПО ИНТЕГРАЛЬНОМУ ИСЧИСЛЕНИЮ).

Для выполнения домашнего задания необходимо, пользуясь табл. 1, заполнить первую строку табл. 2, затем выписать соответствующие вашему номеру варианта данные из табл. 1. Например, Вы учитесь в группе 5, Ваш номер в списке 14. Тогда по табл. 1 имеем:

5	*A*	*C*	*D*	*B*	*K*	*F*	*M*

Вписываем эти буквы в первую строку табл. 2 и выбираем строку, соответствующую четырнадцатому варианту:

Номер по п/п	Коэффициент						
	A	*C*	*D*	*B*	*K*	*F*	*M*
14	4	5	2	-6	-3	7	8

Таблица 1

Значения коэффициентов для разных групп

Группа	Коэффициент						
1	*A*	*B*	*C*	*D*	*K*	*F*	*M*
2	*C*	*D*	*B*	*A*	*K*	*F*	*M*
3	*B*	*A*	*K*	*D*	*C*	*F*	*M*
4	*C*	*A*	*B*	*K*	*D*	*F*	*M*
5	*A*	*C*	*D*	*B*	*K*	*F*	*M*
6	*A*	*K*	*B*	*D*	*C*	*F*	*M*
7	*B*	*K*	*A*	*C*	*D*	*F*	*M*
8	*C*	*K*	*D*	*A*	*B*	*F*	*M*
9	*B*	*D*	*K*	*C*	*A*	*F*	*M*
10	*D*	*K*	*A*	*C*	*B*	*F*	*M*
11	*D*	*C*	*K*	*B*	*A*	*F*	*M*
12	*K*	*C*	*A*	*D*	*B*	*F*	*M*
13	*D*	*A*	*B*	*K*	*C*	*F*	*M*
14	*K*	*B*	*C*	*D*	*A*	*F*	*M*
15	*K*	*A*	*C*	*B*	*D*	*F*	*M*
16	*K*	*C*	*D*	*A*	*B*	*F*	*M*

Таблица 2.

Данные для выполнения домашнего задания

Номер по п/п	Коэффициенты						
1	2	3	–1	5	7	4	14
2	–5	–9	2	1	–4	3	8
3	6	4	1	2	–7	3	12
4	2	–1	6	–9	8	5	13
5	1	5	–4	–3	6	2	–8
6	4	3	11	–1	–4	3	9
7	2	5	1	4	10	3	24
8	5	–2	9	3	–1	4	7
9	–2	7	6	11	–1	4	8
10	–4	10	5	–3	7	2	13
11	–3	2	–4	7	1	4	12
12	–6	5	–1	8	11	2	–6
13	3	–2	9	–5	1	4	17
14	4	5	2	–6	–3	7	8
15	9	3	–5	7	4	3	–12
16	2	5	–1	–3	4	6	–10
17	1	–6	2	3	–5	4	14
18	10	–2	6	–4	3	5	21
19	–4	7	–3	9	6	2	–17
20	2	1	7	12	4	6	–8
21	8	5	–2	4	1	3	17
22	–3	2	–4	6	–7	5	14
23	–1	7	2	5	4	6	3
24	3	–5	6	–4	1	2	8
25	10	–2	4	7	5	3	–27
26	2	11	6	4	–3	5	16
27	1	4	–3	2	9	6	–17
28	4	5	–9	7	3	2	–12
29	3	2	–5	4	7	6	13
30	–2	10	–4	1	–3	4	37

Домашнее задание №4

Задача 1. Вычислить неопределенный интеграл:

1) $\int \frac{Ax+B}{Cx+D}dx$;
2) $\int (Ax+B)^M dx$;
3) $\int \frac{C}{(Dx-B)^M} dx$;
4) $\int \frac{Fdx}{(x-A)(x-B)}$;
5) $\int \sqrt[F]{(Cx+D)^M} dx$;
6) $\int \frac{e^{Ax}}{B+Ce^{Ax}}dx$;
7) $\int \frac{Mx^2+B\ln^F x}{Cx}dx$;
8) $\int C(F)^{Ax+B}dx$;
9) $\int \frac{M\cdot \mathrm{tg}(Cx+D)}{\sin^2(Cx+D)}dx$;
10) $\int \frac{e^{-\left(A^2x^2+B\right)}x}{C}dx$;
11) $\int (F)^x \sqrt{A-B(F)^x}\, dx$;
12) $\int \frac{B\cdot \mathrm{arctg}\,Cx-Kx}{1+C^2x^2}dx$;
13) $\int \frac{\sin(\log_F(Bx))}{Kx}dx$;
14) $\int (Dx+K)e^{-Fx}dx$;
15) $\int \left(Ax^2+Bx+C\right)\sin(Kx)dx$
16) $\int \ln\left(Dx^2+Kx+F\right)dx$;
17) $\int \mathrm{arctg}\sqrt{Mx+K}\,dx$;
18) $\int \frac{Fx+M}{Ax^2+Bx+C}dx$;
19) $\int (B\sin(Mx)+K\cos(Fx))dx$;
20) $\int \cos^F(Ax+K)dx$;
21) $\int \sin(Cx)\cos(Bx)\sin(Kx)dx$;
22) $\int (\mathrm{tg}(Ax)+\mathrm{ctg}(Bx))dx$;
23) $\int \frac{\sin(Fx)}{C+D\sin(Fx)}dx$;
24) $\int \frac{dx}{A-B\cos(Fx)+K\sin(Fx)}$;
25) $\int \frac{Ax^3+Bx^2+Cx+D}{Kx^2+Fx+M}dx$;
26) $\int \frac{M}{\sqrt{Cx^2+Bx+F}}dx$;

27) $\int \frac{A\sqrt[6]{x}-B}{K\sqrt{x}+M\sqrt[3]{x}}dx$;

28) $\int \frac{Cx+D}{\sqrt{Fx^2-Kx-M}}dx$;

29) $\int \frac{dx}{(Ax+B)\sqrt{Kx^2+Mx+F}}$;

30) $\int \sqrt{K^2+Mx-Fx^2}\,dx$.

Пример выполнения домашнего задания №4

Номер п/п	Коэффициенты						
	A	*B*	*C*	*D*	*K*	*F*	*M*
*	–2	–5	1	–3	–4	6	–8

Задача 1. Вычислить неопределенный интеграл:

1) $$\int \frac{-2x-5}{x-3}dx = \int \frac{-2(x-3)-6-5}{x-3}dx = \int\left(-\frac{2(x-3)}{x-3}-\frac{11}{x-3}\right)dx =$$
$$= \int(-2)dx - \int\frac{11dx}{x-3} = -2\int dx - 11\int\frac{dx}{x-3} = -2x - 11\int\frac{d(x-3)}{x-3} =$$
$$= \mathbf{-2x - 11\ln|x-3| + C};$$

2) $$\int(-2x-5)^{-8}dx = \left|\begin{array}{l} t=-2x-5 \\ dt=-2dx \\ dx=-\frac{1}{2}dt \end{array}\right| = \int t^{-8}\left(-\frac{1}{2}dt\right) = -\frac{1}{2}\int t^{-8}dt =$$
$$= -\frac{t^{-8+1}}{2(-8+1)} + C = \frac{1}{14t^7} + C = \mathbf{\frac{1}{14(-2x-5)^7} + C};$$

3) $$\int\frac{dx}{(-3x+5)^{-8}} = \int(5-3x)^8 dx = \left|\begin{array}{l} 5-3x=t \\ -3dx=dt \\ dx=-\frac{1}{3}dt \end{array}\right| = \int t^8\left(-\frac{1}{3}dt\right) = -\frac{1}{3}\int t^8 dt =$$
$$= -\frac{1}{3}\frac{t^{8+1}}{8+1} + C = -\frac{t^9}{27} + C = \mathbf{-\frac{(5-3x)^9}{27} + C};$$

4) $$\int\frac{6dx}{(x+2)(-4x+5)} = 6\int\frac{dx}{(x+2)(-4x+5)} =$$
$$= \left|\begin{array}{l} \frac{1}{(x+2)(-4x+5)} = \frac{A}{x+2} + \frac{B}{-4x+5} = \frac{A(-4x+5)+B(x+2)}{(x+2)(-4x+5)} \\ \text{Приравняем числители:} \\ 1 = A(-4x+5)+B(x+2) \\ \begin{cases} -4A+B=0 \\ 5A+2B=1 \end{cases} \Rightarrow \begin{cases} B=4A \\ 5A+8A=1 \end{cases} \Rightarrow A=\frac{1}{13},\ B=\frac{4}{13} \end{array}\right| =$$
$$= \frac{6}{13}\int\left(\frac{1}{x+2} + \frac{4}{-4x+5}\right)dx = \frac{6}{13}\int\frac{dx}{x+2} + \frac{24}{13}\int\frac{dx}{-4x+5} =$$
$$= \frac{6}{13}\int\frac{d(x+2)}{x+2} + \frac{24}{13}\int\frac{\left(-\frac{1}{4}\right)d(-4x+5)}{-4x+5} =$$
$$= \frac{6}{13}\ln|x+2| - \frac{6}{13}\ln|-4x+5| + C = \mathbf{\frac{6}{13}\ln\left|\frac{x+2}{-4x+5}\right| + C};$$

5) $$\int\sqrt[6]{(x-3)^{-8}}dx = \int(x-3)^{-4/3}dx = \int(x-3)^{-4/3}d(x-3) = \frac{(x-3)^{-4/3+1}}{-4/3+1} + C =$$
$$= \frac{(x-3)^{-1/3}}{-1/3} + C = \mathbf{-\frac{3}{\sqrt[3]{x-3}} + C};$$

6) $\int \frac{e^{-2x}}{-5+e^{-2x}}dx = \left| \begin{array}{l} e^{-2x} = t \\ -2e^{-2x}dx = dt \\ e^{-2x}dx = -\frac{1}{2}dt \end{array} \right| = \int \frac{-\frac{1}{2}dt}{-5+t} = -\frac{1}{2}\int \frac{d(t-5)}{-5+t} =$

$= -\frac{1}{2}\ln|t-5| + C = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{ln}\left|\boldsymbol{e}^{-2x} - \mathbf{5}\right| + \boldsymbol{C}$;

7) $\int \frac{-8x^2 - 5\ln^6 x}{x}dx = \int(-8x)dx - \int \frac{5\ln^6 x}{x}dx = -\frac{8x^2}{2} - 5\int \frac{\ln^6 x}{x}dx =$

$= \left| \begin{array}{l} \ln x = t \\ \frac{1}{x}dx = dt \end{array} \right| = -4x^2 - 5\int t^6 dt = -4x^2 - 5\frac{t^{6+1}}{6+1} + C = -4x^2 - \frac{5}{7}t^7 + C =$

$= -\mathbf{4}\boldsymbol{x}^{\mathbf{2}} - \frac{\mathbf{5}}{\mathbf{7}}\mathbf{ln}^{\mathbf{7}}\boldsymbol{x} + \boldsymbol{C}$;

8) $\int (6)^{-2x-5}dx = \left| \begin{array}{l} -2x-5 = t \\ -2dx = dt \\ dx = -\frac{1}{2}dt \end{array} \right| = \int (6)^t\left(-\frac{1}{2}\right)dt = \left(-\frac{1}{2}\right)\int (6)^t dt =$

$= \left(-\frac{1}{2}\right)\frac{(6)^t}{\ln 6} + C = \left(-\frac{\mathbf{1}}{\mathbf{2}}\right)\frac{\mathbf{(6)^{-2x-5}}}{\mathbf{ln 6}} + \boldsymbol{C}$;

9) $\int \frac{-8\mathrm{tg}(x-3)}{\sin^2(x-3)}dx = -8\int \mathrm{tg}(x-3)\frac{dx}{\sin^2(x-3)} =$

$= -8\int \mathrm{tg}(x-3)(-d(\mathrm{ctg}(x-3))) = 8\int \frac{d(\mathrm{ctg}(x-3))}{\mathrm{ctg}(x-3)} =$

$= |\mathrm{ctg}(x-3) = t| = 8\int \frac{dt}{t}$ = **8 ln|*t*| + *C* = 8 ln|ctg(*x* – 3)| + *C***;

10) $\int e^{-((-2)^2x^2-5)}xdx = \int e^{-4x^2+5}xdx = \left| \begin{array}{l} -4x^2+5 = t \\ -8xdx = dt \\ xdx = -\frac{1}{8}dt \end{array} \right| = \int e^t\left(-\frac{1}{8}\right)dt =$

$= -\frac{1}{8}\int e^t dt = -\frac{1}{8}e^t + C = -\frac{\mathbf{1}}{\mathbf{8}}\boldsymbol{e}^{-4x^2+5} + \boldsymbol{C}$;

11) $\int 6^x\sqrt{-2+5\cdot 6^x}dx = \left| \begin{array}{l} 6^x = t \\ 6^x \ln 6dx = dt \\ 6^x dx = \frac{dt}{\ln 6} \end{array} \right| = \int \sqrt{5\cdot t - 2}\frac{dt}{\ln 6} =$

$= \frac{1}{\ln 6}\int \sqrt{5\cdot t - 2}dt = \left| \begin{array}{l} 5t-2 = u \\ 5dt = du \\ dt = \frac{1}{5}du \end{array} \right| = \frac{1}{\ln 6}\int \sqrt{u}\frac{1}{5}du = \frac{1}{5\ln 6}\cdot\frac{u^{1/2+1}}{1/2+1} + C =$

$$=\frac{1}{5\ln 6}\cdot\frac{u^{3/2}}{3/2}+C=\frac{2}{15\ln 6}\cdot\sqrt{u^3}+C=\frac{2}{15\ln 6}\cdot\sqrt{(5t-2)^3}+C=$$

$$=\mathbf{\frac{2}{15\ln 6}\cdot\sqrt{(5\cdot 6^x-2)^3}+C};$$

12) $$\int\frac{-5\operatorname{arctg}x+4x}{1+x^2}dx=-5\int\frac{\operatorname{arctg}x}{1+x^2}dx+4\int\frac{x}{1+x^2}dx=$$

$$=\left|\begin{matrix}1)\int\frac{\operatorname{arctg}x}{1+x^2}dx=\left|\begin{matrix}\operatorname{atctg}x=t\\ \frac{dx}{1+x^2}=dt\end{matrix}\right|=\int tdt=\frac{t^2}{2}+C=\frac{\operatorname{arctg}^2x}{2}+C\\ 2)\int\frac{x}{1+x^2}dx=\left|\begin{matrix}x^2+1=t\\ 2xdx=dt\\ xdx=\frac{1}{2}dt\end{matrix}\right|=\frac{1}{2}\int\frac{dt}{t}=\frac{1}{2}\ln|t|=\frac{1}{2}\ln\left|x^2+1\right|+C\end{matrix}\right|=$$

$$=-\mathbf{\frac{5}{2}\operatorname{arctg}^2x+2\ln\left|x^2+1\right|+C};$$

13) $$\int\frac{\sin(\log_6(-5x))}{-4x}dx=\left|\begin{matrix}\log_6(-5x)=t\\ \frac{-5}{-5x\ln 6}dx=dt\\ \frac{dx}{x}=\ln 6\cdot dt\end{matrix}\right|=\ln 6\int\frac{\sin t}{-4}dt=$$

$$=-\frac{\ln 6}{4}(-\cos t)+C=\mathbf{\frac{\ln 6}{4}\cos(\log_b(-5x))+C};$$

14) $$\int\underbrace{(-3x-4)}_{U}\underbrace{e^{-6x}dx}_{dV}=\left|\begin{matrix}U=-3x-4 & dV=e^{-6x}dx\\ dU=-3dx & V=\int e^{-6x}dx=\frac{e^{-6x}}{-6}\end{matrix}\right|=$$

$$=(-3x-4)\left(-\frac{e^{-6x}}{6}\right)-\int\frac{e^{-6x}}{-6}(-3dx)=\frac{(3x+4)}{6}e^{-6x}-\frac{1}{2}\int e^{-6x}dx=$$

$$=\frac{(3x+4)}{6}e^{-6x}-\frac{1}{2}\cdot\frac{e^{-6x}}{-6}+C=\mathbf{\frac{(3x+4)}{6}e^{-6x}+\frac{e^{-6x}}{12}+C};$$

15) $$\int(-2x^2-5x+1)\sin(-4x)dx=\int\underbrace{(2x^2+5x-1)}_{U}\underbrace{\sin(4x)dx}_{dV}=$$

$$=\left|\begin{matrix}U=2x^2+5x-1 & dV=\sin(4x)dx\\ dU=(4x+5)dx & V=\int\sin(4x)dx=-\frac{\cos(4x)}{4}\end{matrix}\right|=$$

$$=\left(2x^2+5x-1\right)\left(-\frac{\cos(4x)}{4}\right)-\int\left(-\frac{\cos(4x)}{4}\right)(4x+5)dx=$$

$$=-\frac{2x^2+5x-1}{4}\cos(4x)+\frac{1}{4}\int\underbrace{(4x+5)}_{U}\underbrace{\cos(4x)dx}_{dV}=$$

$$=\left|\begin{matrix}4x+5=U & dV=\cos(4x)dx\\ 4dx=dU & V=\int\cos(4x)dx=\frac{\sin(4x)}{4}\end{matrix}\right|=$$

$$=-\frac{2x^2+5x-1}{4}\cos(4x)+\frac{1}{4}\left((4x+5)\frac{\sin(4x)}{4}-\int\frac{\sin(4x)}{4}4dx\right)=$$

$$= -\frac{2x^2+5x-1}{4}\cos(4x) + \frac{(4x+5)}{16}\sin(4x) - \frac{1}{4}\int \sin(4x)dx =$$

$$= -\frac{\mathbf{2x^2+5x-1}}{\mathbf{4}}\mathbf{\cos(4x)} + \frac{\mathbf{(4x+5)}}{\mathbf{16}}\mathbf{\sin(4x)} + \frac{\mathbf{\cos(4x)}}{\mathbf{16}} + \boldsymbol{C};$$

16) $$\int \underbrace{\ln\left(-3x^2-4x+6\right)}_{U}\underbrace{dx}_{dV} = \left| \begin{matrix} U = \ln\left(-3x^2-4x+6\right) & dV = dx \\ dU = \frac{-6x-4}{-3x^2-4x+6}dx & V = \int dx = x \end{matrix} \right| =$$

$$= x\ln\left(-3x^2-4x+6\right) - \int x\frac{6x+4}{3x^2+4x-6}dx =$$

$$= \left| \begin{matrix} \text{Приведем дробь к правильному виду} \\ \begin{array}{l|l} 6x^2+4x & \underline{3x^2+4x-6} \\ \underline{6x^2+8x-12} & 2 \\ \quad -4x+12 & \end{array} \\ \text{т.е. } \frac{6x^2+4x}{3x^2+4x-6} = 2 + \frac{-4x+12}{3x^2+4x-6} \end{matrix} \right| =$$

$$= x\ln\left(-3x^2-4x+6\right) - \int\left(2 - \frac{4x-12}{3x^2+4x-6}\right)dx =$$

$$= x\ln(-3x^2-4x+6) - \int 2dx + 4\int\frac{x-3}{3x^2+4x-6}dx = \boldsymbol{J}.$$

Для нахождения второго интеграла выделим в числителе и знаменателе производную знаменателя $\left(3x^2+4x-6\right)' = 6x + 4$. Тогда

$$\int\frac{x-3}{3x^2+4x-6}dx = \int\frac{\frac{1}{6}(6x+4) - \frac{4}{6} - 3}{3x^2+4x-6}dx = \frac{1}{6}\int\frac{6x+4}{3x^2+4x-6}dx -$$

$$-\frac{11}{3}\int\frac{dx}{3x^2+4x-6} = \frac{1}{6}\int\frac{d\left(3x^2+4x-6\right)}{3x^2+4x-6}dx - \frac{11}{3}\int\frac{dx}{3\left(x^2+\frac{4}{3}x-2\right)} =$$

$$= \frac{1}{6}\ln\left|3x^2+4x-6\right| - \frac{11}{9}\int\frac{dx}{\left(x+\frac{2}{3}\right)^2 - \frac{4}{9} - 2} = \frac{1}{6}\ln\left|3x^2+4x-6\right| -$$

$$-\frac{11}{9}\int\frac{dx}{\left(x+\frac{2}{3}\right)^2 - \frac{22}{9}} = \frac{1}{6}\ln\left|3x^2+4x-6\right| - \frac{11}{9}\cdot\frac{\sqrt{9}}{2\sqrt{22}}\ln\left|\frac{x+\frac{2}{3}-\sqrt{\frac{22}{9}}}{x+\frac{2}{3}+\sqrt{\frac{22}{9}}}\right| + C =$$

$$= \frac{1}{6}\ln\left|3x^2+4x-6\right| - \frac{\sqrt{22}}{12}\ln\left|\frac{3x+2-\sqrt{22}}{3x+2+\sqrt{22}}\right| + C =$$

$$= \frac{1}{6}\ln\left|3x^2+4x-6\right| - \frac{\sqrt{22}}{12}\ln\left|\frac{3x+2-\sqrt{22}}{3x+2+\sqrt{22}}\right| + C.$$

Тогда:

$$J = x\ln\left(-3x^2-4x+6\right) - 2x + \frac{2}{3}\ln\left|3x^2+4x-6\right| - \frac{\sqrt{22}}{3}\ln\left|\frac{3x+2-\sqrt{22}}{3x+2+\sqrt{22}}\right| + C;$$

17) $\int \underbrace{\operatorname{arctg}\sqrt{-8x-4}}_{U}\underbrace{dx}_{dV}=$

$$=\left|\begin{matrix} U=\operatorname{arctg}\sqrt{-8x-4} & dV=dx \\ dU=\dfrac{-8}{1+\left(\sqrt{-8x-4}\right)^2}\cdot\dfrac{1}{2\sqrt{-8x-4}}\,dx & V=\int dx=x\end{matrix}\right|=$$

$$=x\cdot\operatorname{arctg}\sqrt{-8x-4}-\int x\left(\frac{-4dx}{(1-8x-4)\sqrt{-8x-4}}\right)dx=$$

$$=x\cdot\operatorname{arctg}\sqrt{-8x-4}-4\int\frac{xdx}{(8x+3)\sqrt{-8x-4}}=\left|\begin{matrix}\sqrt{-8x-4}=t \\ -8x-4=t^2 \\ x=\dfrac{-4-t^2}{8} \\ dx=-\dfrac{2tdt}{8}=-\dfrac{1}{4}tdt\end{matrix}\right|=$$

$$=x\cdot\operatorname{arctg}\sqrt{-8x-4}-4\int\frac{\dfrac{-4-t^2}{8}\left(-\dfrac{1}{4}tdt\right)}{(-4-t^2+3)t}=x\cdot\operatorname{arctg}\sqrt{-8x-4}+$$

$$+\int\frac{\left(4+t^2\right)dt}{8(t^2+1)}=x\cdot\operatorname{arctg}\sqrt{-8x-4}+\frac{1}{8}\int\frac{t^2+1+3}{t^2+1}dt=x\cdot\operatorname{arctg}\sqrt{-8x-4}+$$

$$+\frac{1}{8}\int\left(1+\frac{3}{t^2+1}\right)dt=x\cdot\operatorname{arctg}\sqrt{-8x-4}+\frac{1}{8}\int dt+\frac{3}{8}\int\frac{dt}{t^2+1}=$$

$$=x\cdot\operatorname{arctg}\sqrt{-8x-4}+\frac{1}{8}t+\frac{3}{8}\operatorname{arctg}t+C=$$

$$=\boldsymbol{x\cdot\operatorname{arctg}\sqrt{-8x-4}+\frac{1}{8}\sqrt{-8x-4}+\frac{3}{8}\operatorname{arctg}\sqrt{-8x-4}+C};$$

18) $\int\dfrac{6x-8}{-2x^2-5x+1}dx=-\int\dfrac{6x-8}{2x^2+5x-1}dx=$

$$=\left|\begin{matrix}\text{Выделим в числителе производную знаменателя} \\ \left(2x^2+5x-1\right)'=4x+5 \quad\Rightarrow\quad d\left(2x^2+5x-1\right)=(4x+5)dx\end{matrix}\right|=$$

$$=-\int\frac{\dfrac{3}{2}(4x+5)-\dfrac{15}{2}-8}{2x^2+5x-1}dx=-\frac{3}{2}\int\frac{4x+5}{2x^2+5x-1}dx+\frac{31}{2}\int\frac{dx}{2x^2+5x-1}=$$

$$=-\frac{3}{2}\int\frac{d\left(2x^2+5x-1\right)}{2x^2+5x-1}+\frac{31}{4}\int\frac{dx}{x^2+\dfrac{5}{2}x-\dfrac{1}{2}}=-\frac{3}{2}\ln\left|2x^2+5x-1\right|+$$

$$+\frac{31}{4}\int\frac{dx}{\left(x+\dfrac{5}{4}\right)^2-\dfrac{25}{16}-\dfrac{1}{2}}=-\frac{3}{2}\ln\left|2x^2+5x-1\right|+\frac{31}{4}\int\frac{dx}{\left(x+\dfrac{5}{4}\right)^2-\dfrac{33}{16}}=$$

$$=-\frac{3}{2}\ln\left|2x^2+5x-1\right|+\frac{31}{4}\cdot\frac{1}{2\sqrt{\dfrac{33}{16}}}\cdot\ln\left|\frac{x+\dfrac{5}{4}-\sqrt{\dfrac{33}{16}}}{x+\dfrac{5}{4}+\sqrt{\dfrac{33}{16}}}\right|+C=$$

$$=\boldsymbol{-\frac{3}{2}\ln\left|2x^2+5x-1\right|+\frac{31}{2\sqrt{33}}\cdot\ln\left|\frac{4x+5-\sqrt{33}}{4x+5+\sqrt{33}}\right|+C};$$

19) $\int(-5\sin(-8x)-4\cos(6x))dx=\int(5\sin(8x)-4\cos(6x))dx=$

$$=5\int\sin(8x)dx-4\int\cos(6x)dx=\frac{5}{8}\int\sin(8x)d(8x)-\frac{4}{6}\int\cos(6x)d(6x)=$$

$$=-\mathbf{\frac{5}{8}\cos(8x)-\frac{2}{3}\sin(6x)+C};$$

20) $\int\cos^6(-2x-4)dx=\left|\begin{array}{l}t=-2x-4\\ dt=-2dx\\ dx=-\frac{1}{2}dt\end{array}\right|=-\frac{1}{2}\int\cos^6 tdt=-\frac{1}{2}\int\left(\frac{1+\cos(2t)}{2}\right)^3dt=$

$$=-\frac{1}{2}\cdot\frac{1}{8}\int\left(1+3\cos(2t)+3\cos^2(2t)+\cos^3(2t)\right)dt=$$

$$=-\frac{1}{16}\left(\int dt+3\int\cos(2t)dt+3\int\cos^2(2t)dt+\int\cos^3(2t)dt\right)=$$

$$=-\frac{1}{16}\left(t+3\int\cos(2t)\frac{d(2t)}{2}+3\int\frac{(1+\cos(4t))}{2}dt+\int\cos^3(2t)\frac{d(2t)}{2}\right)=$$

$$=-\frac{1}{16}\left(t+\frac{3}{2}\sin 2t+\frac{3}{2}\int dt+\frac{3}{2}\int\cos(4t)\frac{d(4t)}{4}+\int\cos^2(2t)\frac{d\sin(2t)}{2}\right)=$$

$$=-\frac{1}{16}\left(t+\frac{3}{2}\sin 2t+\frac{3}{2}t+\frac{3}{8}\sin(4t)+\frac{1}{2}\int\left(1-\sin^2(2t)\right)d\sin(2t)\right)=$$

$$=|\sin 2t=u|=-\frac{1}{16}\left(\frac{5}{2}t+\frac{3}{2}\sin 2t+\frac{3}{8}\sin(4t)+\frac{1}{2}\int\left(1-u^2\right)du\right)=$$

$$=-\frac{1}{16}\left(\frac{5}{2}t+\frac{3}{2}\sin 2t+\frac{3}{8}\sin(4t)+\frac{1}{2}u-\frac{1}{2}\frac{u^3}{3}+C\right)=-\frac{1}{16}\left(\frac{5}{2}(-2x-4)+\right.$$

$$\left.+\frac{3}{2}\sin(-4x-8)+\frac{3}{8}\sin(-8x-16)+\frac{\sin(2t)}{2}-\frac{1}{2}\frac{\sin^3(2t)}{3}+C\right)=$$

$$=-\frac{1}{16}\left(-5x-10-\frac{3}{2}\sin(4x+8)-\frac{3}{8}\sin(8x+16)-\frac{\sin(4x+8)}{2}-\right.$$

$$\left.-\frac{\sin^3(-4x-8)}{6}+C\right)=-\mathbf{\frac{1}{16}\left(-5x-10-2\sin(4x+8)-\frac{3}{8}\sin(8x+16)+\right.}$$

$$\mathbf{\left.+\frac{\sin^3(4x+8)}{6}+C\right)};$$

21) $\int\sin x\cdot\cos(-5x)\cdot\sin(-4x)dx=-\int\sin x\cdot\cos 5x\cdot\sin 4xdx=$

$$=-\int\frac{1}{2}\left(\sin(x-5x)+\sin(x+5x)\right)\cdot\sin(4x)dx=$$

$$=-\frac{1}{2}\int\left(\sin(6x)\sin(4x)-\sin^2(4x)\right)dx=$$

$$=-\frac{1}{2}\int\frac{1}{2}\left(\cos(2x)-\cos(10x)\right)dx+\frac{1}{2}\int\frac{1-\cos(8x)}{2}dx=$$

$$=-\frac{1}{4}\int\cos(2x)dx+\frac{1}{4}\int\cos(10x)dx+\frac{1}{4}\int dx-\frac{1}{4}\int\cos(8x)dx=$$

$$=-\mathbf{\frac{1}{8}\sin 2x+\frac{1}{40}\sin(10x)+\frac{1}{4}x-\frac{1}{32}\sin 8x+C};$$

22) $\int\left(\mathrm{tg}(-2x)+\mathrm{ctg}(-5x)\right)dx=-\int\left(\mathrm{tg}(2x)+\mathrm{ctg}(5x)\right)dx=$

$$=-\int\frac{\sin 2x}{\cos 2x}dx-\int\frac{\cos 5x}{\sin 5x}dx=-\frac{1}{2}\int\frac{\sin 2x}{\cos 2x}d(2x)-\frac{1}{5}\int\frac{\cos 5x}{\sin 5x}d(5x)=$$

$$=\frac{1}{2}\int\frac{d(\cos 2x)}{\cos 2x}-\frac{1}{5}\int\frac{d(\sin 5x)}{\sin 5x}=\boldsymbol{\frac{1}{2}\ln|\cos 2x|-\frac{1}{5}\ln|\sin 5x|+C};$$

23) $\int\frac{\sin 6x\,dx}{1-3\sin 6x}=\frac{1}{6}\int\frac{\sin 6x\,d(6x)}{1-3\sin 6x}=|U=6x|=\frac{1}{6}\int\frac{\sin U\,dU}{1-3\sin U}=$

$$=\left|\begin{matrix}\mathrm{tg}\frac{U}{2}=t\\ \sin U=\frac{2t}{1+t^2}\\ dU=\frac{2dt}{1+t^2}\end{matrix}\right|=\frac{1}{6}\int\frac{\frac{2t}{1+t^2}\cdot\frac{2dt}{1+t^2}}{1-\frac{6t}{1+t^2}}=\frac{2}{3}\int\frac{t\,dt}{\left(1+t^2\right)\left(1+t^2-6t\right)}=$$

$$=\left|\begin{matrix}\frac{t}{\left(t^2+1\right)\left(t^2-6t+1\right)}=\frac{At+B}{\left(t^2+1\right)}+\frac{Ct+E}{\left(t^2-6t+1\right)}=\frac{(At+B)\left(t^2-6t+1\right)+(Ct+E)\left(t^2+1\right)}{\left(t^2+1\right)\left(t^2-6t+1\right)}\\ \text{Приравняем числители:}\\ t=At^3-6At^2+At+Bt^2-6Bt+B+Ct^3+Et^2+Ct+E\\ \begin{matrix}t^3:\\ t^2:\\ t:\\ t^0:\end{matrix}\begin{cases}A+C=0\\ -6A+B+E=0\\ A-6B+C=1\\ B+E=0\end{cases}\begin{cases}A=-C\\ 6C-E+E=0\\ -C+6E+C=1\\ B=-E\end{cases}\begin{cases}A=-C=0\\ C=0\\ E=\frac{1}{6}\\ B=-\frac{1}{6}\end{cases}\\ \frac{t}{\left(t^2+1\right)\left(t^2-6t+1\right)}=\frac{1}{6}\left(\frac{1}{\left(t^2-6t+1\right)}-\frac{1}{\left(t^2+1\right)}\right)\end{matrix}\right|=$$

$$=\frac{2}{3}\cdot\frac{1}{6}\left(\int\frac{dt}{\left(t^2-6t+1\right)}-\int\frac{dt}{\left(t^2+1\right)}\right)=\frac{1}{9}\left(\int\frac{dt}{\left((t-3)^2-9+1\right)}-\mathrm{atctg}\,t\right)=$$

$$=\frac{1}{9}\left(\int\frac{dt}{\left((t-3)^2-8\right)}-\mathrm{atctg}\,t\right)=\frac{1}{9}\left(\frac{1}{2\sqrt{8}}\ln\left|\frac{t-3-\sqrt{8}}{t-3+\sqrt{8}}\right|-\mathrm{atctg}\,t\right)+C=$$

$$=\left|t=\mathrm{tg}\frac{U}{2}=\mathrm{tg}(3x)\right|=\boldsymbol{\frac{1}{9}\left(\frac{1}{4\sqrt{2}}\ln\left|\frac{\mathrm{tg}(3x)-3-\sqrt{8}}{\mathrm{tg}(3x)-3+\sqrt{8}}\right|-3x\right)+C};$$

24) $\int\frac{dx}{-2+5\cos 6x+4\sin(-6x)}=\int\frac{dx}{-2+5\cos 6x-4\sin 6x}=\left|\begin{matrix}6x=U\\ 6dx=du\\ dx=\frac{1}{6}dU\end{matrix}\right|=$

$$=\frac{1}{6}\int\frac{dU}{-2+5\cos U-4\sin U}=\left|\begin{matrix}\mathrm{tg}\frac{U}{2}=t;\ \sin U=\frac{2t}{1+t^2}\\ \cos U=\frac{1-t^2}{1+t^2};\ dU=\frac{2dt}{1+t^2}\end{matrix}\right|=$$

$$=\frac{1}{6}\int\frac{\frac{2dt}{1+t^2}}{-2+5\frac{1-t^2}{1+t^2}-4\frac{2t}{1+t^2}}=\frac{1}{3}\int\frac{dt}{-2-2t^2+5-5t^2-8t}=$$

$$=\frac{1}{3}\int\frac{dt}{-7t^2-8t+3}=-\frac{1}{21}\int\frac{dt}{t^2+\frac{8}{7}t-\frac{3}{7}}=-\frac{1}{21}\int\frac{dt}{\left(t+\frac{4}{7}\right)^2-\frac{16}{49}-\frac{3}{7}}=$$

$$=-\frac{1}{21}\int\frac{dt}{\left(t+\frac{4}{7}\right)^2-\frac{37}{49}}=-\frac{1}{21}\cdot\frac{1}{2\sqrt{\frac{37}{49}}}\ln\left|\frac{t+\frac{4}{7}-\sqrt{\frac{37}{49}}}{t+\frac{4}{7}+\sqrt{\frac{37}{49}}}\right|+C=$$

$$=-\frac{1}{6\sqrt{37}}\ln\left|\frac{7t+4-\sqrt{37}}{7t+4+\sqrt{37}}\right|+C=\left|t=\mathrm{tg}\frac{U}{2}=\mathrm{tg}(3x)\right|=$$

$$=-\frac{\mathbf{1}}{\mathbf{6\sqrt{37}}}\mathbf{ln}\left|\frac{\mathbf{7tg(3x)+4-\sqrt{37}}}{\mathbf{7tg(3x)+4+\sqrt{37}}}\right|+\boldsymbol{C};$$

25) $\int\frac{-2x^3-5x^2+x-3}{-4x^2+6x-8}dx=\int\frac{2x^3+5x^2-x+3}{4x^2-6x+8}dx=J.$

Приведем дробь к правильному виду

$$\begin{array}{r|l} 2x^3+5x^2-x+3 & 4x^2-6x+8 \\ \underline{2x^3-3x^2+4x} & \frac{1}{2}x+2 \\ 8x^2-5x+3 & \\ \underline{8x^2-12x+16} & \\ 7x-13, & \end{array}$$

т. е. $\frac{2x^3+5x^2-x+3}{4x^2-6x+8}=\frac{1}{2}x+2+\frac{7x-13}{4x^2-6x+8}$.

Тогда

$$J=\int\left(\frac{1}{2}x+2+\frac{7x-13}{4x^2-6x+8}\right)dx=\frac{1}{2}\int xdx+2\int dx+\frac{1}{2}\int\frac{7x-13}{2x^2-3x+4}dx=$$

$$=\frac{1}{2}\int xdx+2\int dx+\frac{1}{2}\int\frac{7x-13}{2x^2-3x+4}dx=J.$$

В третьем интеграле выделим в числителе производную знаменателя
$(2x^2-3x+4)'=4x-3$, т.е.
$d(2x^2-3x+4)=(4x-3)dx.$

Тогда

$$J=\frac{1}{2}\frac{x^2}{2}+2x+\frac{1}{2}\int\frac{\frac{7}{4}(4x-3)+\frac{21}{4}-13}{2x^2-3x+4}dx=\frac{x^2}{4}+2x+$$

$$+\frac{7}{8}\int\frac{d\left(2x^2-3x+4\right)}{2x^2-3x+4}dx-\frac{31}{8}\int\frac{dx}{2x^2-3x+4}=\frac{x^2}{4}+2x+\frac{7}{8}\int\frac{d\left(2x^2-3x+4\right)}{2x^2-3x+4}dx-$$

$$-\frac{31}{16}\int\frac{dx}{x^2-\frac{3}{2}x+2}=\frac{x^2}{4}+2x+\frac{7}{8}\ln\left|2x^2-3x+4\right|-\frac{31}{16}\int\frac{dx}{\left(x-\frac{3}{4}\right)^2-\frac{9}{16}+2}=$$

$$= \frac{x^2}{4} + 2x + \frac{7}{8}\ln\left|2x^2 - 3x + 4\right| - \frac{31}{16}\int \frac{dx}{\left(x - \frac{3}{4}\right)^2 + \frac{23}{16}} = \frac{x^2}{4} + 2x +$$

$$+ \frac{7}{8}\ln\left|2x^2 - 3x + 4\right| - \frac{31}{16}\cdot\frac{1}{\sqrt{\frac{23}{16}}}\cdot \operatorname{arctg}\frac{x - \frac{3}{4}}{\sqrt{\frac{23}{16}}} + C = \frac{x^2}{4} + 2x + \frac{7}{8}\ln\left|2x^2 - 3x + 4\right| -$$

$$- \frac{31}{16}\cdot\frac{4}{\sqrt{23}}\cdot \operatorname{arctg}\frac{4x-3}{\sqrt{23}} + C =$$

$$= \mathbf{\frac{x^2}{4} + 2x + \frac{7}{8}\ln\left|2x^2 - 3x + 4\right| - \frac{31}{4\sqrt{23}}\cdot \operatorname{arctg}\frac{4x-3}{\sqrt{23}} + C}\ ;$$

26) $\int \frac{-8dx}{\sqrt{x^2 - 5x + 6}} = -8\int \frac{dx}{\sqrt{x^2 - 5x + 6}} = -8\int \frac{dx}{\sqrt{\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 6}} =$

$$= -8\int \frac{dx}{\sqrt{\left(x - \frac{5}{2}\right)^2 - \frac{1}{4}}} = -8\ln\left|x - \frac{5}{2} + \sqrt{\left(x - \frac{5}{2}\right)^2 - \frac{1}{4}}\right| + C =$$

$$= \mathbf{-8\ln\left|x - \frac{5}{2} + \sqrt{x^2 - 5x + 6}\right| + C}\ ;$$

27) $\int \frac{-2\sqrt[6]{x} + 5}{-4\sqrt{x} - 8\sqrt[3]{x}}\,dx = \int \frac{2\sqrt[6]{x} - 5}{4\sqrt{x} + 8\sqrt[3]{x}}\,dx = \left| \begin{array}{l} \sqrt[6]{x} = t \\ x = t^6 \\ dx = 6t^5 dt \end{array} \right| = \int \frac{2t-5}{4t^3 + 8t^2}\,6t^5 dt =$

$$= \frac{3}{2}\int \frac{t^5(2t-5)}{t^2(t+2)}\,dt = \frac{3}{2}\int \frac{t^3(2t-5)}{t+2}\,dt = \frac{3}{2}\int \frac{2t^4 - 5t^3}{t+2}\,dt = \boldsymbol{J}.$$

Приведем дробь к правильному виду:

$$\begin{array}{l|l} 2t^4 - 5t^3 & t + 2 \\ \underline{2t^4 + 4t^3} & 2t^3 - 9t^2 + 18t - 36 \\ \quad -9t^3 & \\ \quad \underline{-9t^3 - 18t^2} & \\ \qquad 18t^2 & \\ \qquad \underline{18t^2 + 36t} & \\ \qquad\quad -36t & \\ \qquad\quad \underline{-36t - 72} & \\ \qquad\qquad 72 & \end{array}$$

т. е. $\frac{2t^4 - 5t^3}{t+2} = 2t^3 - 9t^2 + 18t - 36 + \frac{72}{t+2}$.

Тогда:

$$J = \frac{3}{2}\int\left(2t^3 - 9t^2 + 18t - 36 + \frac{72}{t+2}\right)dt = \frac{3}{2}\left(\frac{2t^4}{4} - \frac{9t^3}{3} + \right.$$

$$\left. + \frac{18t^2}{2} - 36t + 72\ln|t+2|\right) + C = \frac{3t^4}{4} - \frac{9t^3}{2} + \frac{27t^2}{2} - 54t + 108\ln|t+2| + C =$$

$$=\frac{3}{4}x^{\frac{2}{3}}-\frac{9}{2}x^{\frac{1}{2}}+\frac{27}{2}x^{\frac{1}{3}}-54x^{\frac{1}{6}}+108\ln\left|x^{\frac{1}{6}}+2\right|+C=$$

$$=\mathbf{\frac{3}{4}\sqrt[3]{x^2}-\frac{9}{2}\sqrt{x}+\frac{27}{2}\sqrt[3]{x}-54\sqrt[6]{x}+108\ln\left|\sqrt[6]{x}+2\right|+C};$$

28) $$\int\frac{x-3}{\sqrt{6x^2+4x+8}}dx=\left|\begin{array}{l}\text{Выделим в числителе}\\ \text{производную знаменателя}\\ \left(6x^2+4x+8\right)'=12x+4\\ d\left(6x^2+4x+8\right)=(12x+4)dx\end{array}\right|=$$

$$=\int\frac{\frac{1}{12}(12x+4)-\frac{4}{12}-3}{\sqrt{6x^2+4x+8}}dx=\frac{1}{12}\int\frac{d\left(6x^2+4x+8\right)}{\sqrt{6x^2+4x+8}}-\frac{10}{3}\int\frac{dx}{\sqrt{6x^2+4x+8}}=$$

$$=\frac{1}{12}\frac{\left(6x^2+4x+8\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}-\frac{10}{3\sqrt{6}}\int\frac{dx}{\sqrt{x^2+2/3\,x+4/3}}=$$

$$=\frac{1}{6}\sqrt{6x^2+4x+8}-\frac{10}{3\sqrt{6}}\int\frac{dx}{\sqrt{\left(x+1/3\right)^2-1/9+4/3}}=$$

$$=\frac{1}{6}\sqrt{6x^2+4x+8}-\frac{10}{3\sqrt{6}}\int\frac{dx}{\sqrt{\left(x+1/3\right)^2+11/9}}=$$

$$=\frac{1}{6}\sqrt{6x^2+4x+8}-\frac{10}{3\sqrt{6}}\ln\left|x+1/3+\sqrt{\left(x+1/3\right)^2+11/9}\right|+C=$$

$$=\mathbf{\frac{1}{6}\sqrt{6x^2+4x+8}-\frac{10}{3\sqrt{6}}\ln\left|x+1/3+\sqrt{x^2+2/3\,x+4/3}\right|+C};$$

29) $$\int\frac{dx}{(-2x-5)\sqrt{-4x^2-8x+6}}=-\int\frac{dx}{(2x+5)\sqrt{-4x^2-8x+6}}=$$

$$\left|\begin{array}{l}\frac{1}{2x+5}=t;\ 2x+5=\frac{1}{t}\\ 2x=\frac{1}{t}-5;\ x=\frac{1}{2t}-\frac{5}{2}\\ dx=-\frac{1}{2t^2}dt\end{array}\right|=-\int\frac{-\frac{1}{2t^2}dt}{\frac{1}{t}\sqrt{-4\left(\frac{1}{2t}-\frac{5}{2}\right)^2-8\left(\frac{1}{2t}-\frac{5}{2}\right)+6}}=$$

$$=\frac{1}{2}\int\frac{dt}{t\sqrt{-4\left(\frac{1}{4t^2}-\frac{5}{2t}+\frac{25}{4}\right)-\frac{8}{2t}+20+6}}=$$

$$=\frac{1}{2}\int\frac{dt}{t\sqrt{\frac{-1+6t+t^2}{t^2}}}=\frac{1}{2}\int\frac{dt}{\sqrt{-1+6t+t^2}}=\frac{1}{2}\int\frac{dt}{\sqrt{(t+3)^2-9-1}}=$$

$$=\frac{1}{2}\int\frac{dt}{\sqrt{(t+3)^2-10}}=\frac{1}{2}\ln\left|t+3+\sqrt{(t+3)^2-10}\right|+C=$$

$$=\frac{1}{2}\ln\left|\frac{1}{2x+5}+3+\sqrt{\left(\frac{1}{2x+5}\right)^2+\frac{6}{2x+5}-1}\right|+C=$$

$$=\frac{1}{2}\ln\left|\frac{1+6x+15+\sqrt{1+6(2x+5)-(2x+5)^2}}{2x+5}\right|+C=$$

$$=\frac{1}{2}\ln\left|\frac{16+6x+\sqrt{-4x^2-20x-25+12x+30+1}}{2x+5}\right|+C=$$

$$=\mathbf{\frac{1}{2}\ln\left|\frac{16+6x+\sqrt{-4x^2-8x+6}}{2x+5}\right|+C};$$

30) $$\int\sqrt{16-8x-6x^2}\,dx=\sqrt{6}\int\sqrt{\frac{16}{6}-\frac{8}{6}x-x^2}\,dx=\sqrt{6}\int\sqrt{\frac{8}{3}-\frac{4}{3}x-x^2}\,dx=$$

$$=\sqrt{6}\int\sqrt{\frac{8}{3}-\left(\frac{4}{3}x+x^2\right)}dx=\sqrt{6}\int\sqrt{\frac{8}{3}-\left(\left(x+\frac{2}{3}\right)^2-\frac{4}{9}\right)}dx=$$

$$=\sqrt{6}\int\sqrt{\frac{8}{3}+\frac{4}{9}-\left(x+\frac{2}{3}\right)^2}\,dx=\left|\begin{matrix}x+\frac{2}{3}=t\\ dx=dt\end{matrix}\right|=\sqrt{6}\int\sqrt{\frac{28}{9}-t^2}\,dt=$$

$$=\left|\begin{matrix}t=\frac{\sqrt{28}}{3}\sin U\\ dt=\frac{\sqrt{28}}{3}\cos U dU\end{matrix}\right|=\sqrt{6}\int\sqrt{\frac{28}{9}-\frac{28}{9}\sin^2 U}\,\frac{\sqrt{28}}{3}\cos U dU=$$

$$=\sqrt{6}\,\frac{\sqrt{28}}{3}\,\frac{\sqrt{28}}{3}\int\cos^2 U dU=\frac{28\sqrt{6}}{9}\int\frac{1+\cos 2U}{2}dU=$$

$$=\frac{14\sqrt{6}}{9}\left(U+\frac{\sin 2U}{2}\right)+C=\left|\begin{matrix}\sin U=\frac{3}{\sqrt{28}}t;\ \ U=\arcsin\left(\frac{3}{\sqrt{28}}t\right)\\ \sin 2U=2\sin U\cos U=\frac{6}{\sqrt{28}}t\sqrt{1-\frac{9}{28}t^2}\end{matrix}\right|=$$

$$=\frac{14\sqrt{6}}{9}\arcsin\frac{3t}{\sqrt{28}}+\frac{7\sqrt{6}}{9}\,\frac{6}{\sqrt{28}}t\sqrt{\frac{28-9t^2}{28}}+C=\left|t=x+\frac{2}{3}\right|=$$

$$=\frac{14\sqrt{6}}{9}\arcsin\frac{3x+2}{\sqrt{28}}+\frac{7\sqrt{6}}{3}\,\frac{2}{28}\left(x+\frac{2}{3}\right)\sqrt{28-9\left(x+\frac{2}{3}\right)^2}+C=$$

$$=\frac{14\sqrt{6}}{9}\arcsin\frac{3x+2}{\sqrt{28}}+\frac{7\sqrt{6}}{3\cdot 14}\left(\frac{3x+2}{3}\right)\sqrt{28-9x^2-12x-4}+C=$$

$$=\frac{14\sqrt{6}}{9}\arcsin\frac{3x+2}{\sqrt{28}}+\frac{\sqrt{6}}{18}(3x+2)\sqrt{-9x^2-12x+24}+C=$$

$$=\mathbf{\frac{14\sqrt{6}}{9}\arcsin\frac{3x+2}{\sqrt{28}}+\frac{(3x+2)\sqrt{-6x^2-8x+16}}{6}+C}.$$